B.E. (Mechanical Engineering) Fourth Semester (C.B.S.)

Applied Mathematics - IV

P. Pages : 3 **NIR/KW/18/3368**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.
10. Table for area under standard normal curve is permitted.

1. a) Using Regula Falsi Method, find the root of the equation 3x = 1+ cosx correct to third decimal place. **6**

b) Apply Crout's Method to solve the equations **6**

$3x + 2y + 7z = 4$
$2x + 3y + z = 5$
$3x + 4y + z = 7$

OR

2. a) Find the root of equation **6**

$X \log_{10}^{x} - 2 = 0$ by Newton - Raphson Method correct upto three places of decimal.

b) Apply Gauss Seidel Method to solve the equations **6**

$20x + y - 2z = 17$
$3x + 20y - z = -18$
$2x - 3y + 20z = 25$

3. a) Solve by using modified Euler's Method **7**

$\frac{dy}{dx} = 2 + \sqrt{xy}$,given y = 1, when x = 1 find y (1.4) where h = 0.2

b) Use Milne's predictor - corrector Method to find y (0.4) from the D.E. $\frac{dy}{dx} = 1 + xy^2$, **7**

y (0) = 1, y (0.1) = 1.105, y (0.2) = 1.223, y (0.3) = 1.355

OR

4. a) Solve $\frac{d^2y}{dx^2} = xy - 4y,\ y(0) = 3,\ y^1(0) = 0$ for $x = 0.1$ by Runge Kutta Fourth order Method. **7**

b) Find the largest eigen value and the corresponding eigen vector for the Matrix **7**

$$A = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}$$

5. a) Prove that $Z\left\{n^p\right\} = -z\dfrac{d}{dz}\, Z\left\{n^{p-1}\right\}$, where P is any positive integer and hence deduce that $Z\{n\} = \dfrac{z}{(z-1)^2}$ **6**

b) Show that $Z^{-1}\left\{e^{2/z}\right\} = \dfrac{2^n}{n!}$ by using convolution theorem. **6**

OR

6. a) Use Z-Transform to solve the difference equation **6**

$$y_{n+2} + 5y_{n+1} + 6y_n = 6^n,\ y_o = o,\ y_1 = 1$$

b) Find $Z^{-1}\left\{\dfrac{16Z^3}{(4z-1)^2\,(z-1)}\right\}$ by residue Method **6**

7. a) Solve by Frobenius Method the equation **8**

$$4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

b) Prove that **6**

$$4J_0'''(x) + 3J_0'(x) + J_3(x) = 0$$

OR

8. a) Show that **7**

i) $P_n(-x) = (-1)^n P_n(x)$

ii) $P_n(-1) = (-1)^n$

b) If **7**

$$f(x) = \begin{cases} 0, & -1 < x < 0 \\ 1, & 0 < x < 1, \end{cases}$$

Obtain Legendre's expansion for f(x)

9. a) Find the distribution function for the random variable X having density function **7**

$$f(x) = \begin{cases} x^2/9, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

and hence find $P(1 \le x \le 2)$

b) Let X be a random Variable giving the number of aces in a random draw of four cards from a pack of 52 cards. Find the probability function and the distribution function for X **7**

OR

10. a) The joint probability function of two random variable x and y is given by **7**

$$f(x,y) = \begin{cases} \frac{x+y}{27}, & x = 0,1,2,\ y = 0,1,2 \\ 0, & \text{otherwise} \end{cases}$$

Find conditional probability function of y given x and x given y.

b) A random variable x has density function given by **7**

$$f(x) = \begin{cases} 2\,c^{-2x}, & x > 0 \\ 0, & x < 0 \end{cases}$$

Find

i) $E(x)$

ii) $\mathrm{Var}(x)$

iii) $E\left[(x-1)^2\right]$

iv) Moment generating function

11. a) Define exponential distribution and find its mean, variance and Moment generating function. **7**

b) Show that the Poisson distribution is the limiting form of the Binomial distribution when P(orq) is very small and n large enough. **7**

OR

12. a) If the diameters of ball bearings are normally distributed with mean 15.60 mm. and standard deviation 0.06 mm, determine the percentage of ball bearing with diameters **8**

a) Between 15.50 and 15.70 mm
b) Greater than 15.70 mm
c) Less than 15.40 mm.
d) Equal to 15.60 mm.

b) Let X be uniformly distributed in $-2 \le x \le 2$ **6**

Find

i) $P(x<1)$

ii) $P\left(|x-1| \ge \frac{1}{2}\right)$

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