

B.E.(Electronics Engineering / Electrical Engineering (Electronics & Power) /
Electronics Telecommunication / Electronics Communication /
Mechanical Engineering) Semester Third (C.B.S.)

Applied Mathematics - III

Paper - I

P. Pages : 3

Time : Three Hours



KNT/KW/16/7212/7217/7222/7227

Max. Marks : 80

- Notes :
1. All questions carry marks as indicated. Attempt **six** questions as follows :
 2. Question 1 OR Questions No. 2.
 3. Question 3 OR Questions No. 4.
 4. Question 5 OR Questions No. 6.
 5. Question 7 OR Questions No. 8.
 6. Question 9 OR Questions No. 10.
 7. Question 11 OR Questions No. 12.
 8. Assume suitable data whenever necessary.
 9. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\} = \bar{f}(s)$, then prove that

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} \bar{f}(s). \text{ Hence find } L[t \sin 2t].$$

- b) Find $L^{-1}\left\{\frac{S^2}{(S^2+4)(S^2+9)}\right\}$ by convolution theorem.

OR

2. a) If $f(t) = \cos 2t$, $\pi < t < 2\pi$ and $f(t) = 0$, otherwise, express $f(t)$ in terms of unit step function and find its Laplace transform.

- b) Solve $\frac{d^2x}{dt^2} + 9x = \cos(2t)$, given $x(0) = 1$, $x\left(\frac{\pi}{2}\right) = -1$, using Laplace transform technique.

3. a) Find Fourier Series for $f(x) = x - x^2$ in interval $-1 < x < 1$.

- b) Find Fourier Sine transform of $e^{-|x|}$ and hence show that :

$$\int_0^{\infty} \frac{x \sin(mx)}{1+x^2} dx = \frac{\pi}{2} e^{-m}, m > 0$$

OR

4. a) Obtain half range sine series for $f(x) = \pi x - x^2$ in the interval $(0, \pi)$. 6
- b) Find Fourier transform of 6
- $$f(x) = \begin{cases} 1, & \text{for } |x| < 1 \\ 0, & \text{for } |x| > 1, \end{cases}$$
- hence find $\int_0^{\infty} \frac{\sin x}{x} dx$.

5. Find the extremals of 6
- $$V(y) = \int_{x_0}^{x_1} \left\{ (y'')^2 - 2(y')^2 + y^2 - 2y \sin x \right\} dx$$

OR

6. Find the curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about x axis gives minimum surface area. 6
7. a) Prove that $u = e^{-x} [x \sin y - y \cos y]$ is harmonic. Find v such that $f(z) = u + iv$ is analytic. 6
- b) If $f(a) = \oint_C \frac{3Z^2 + 7Z + 1}{Z - a} dz$, where C is a circle $|Z| = 2$, find values of 6
- i) $f(3)$
 ii) $f'(1-i)$
 iii) $f''(1-i)$
- c) Expand the function $f(Z) = (Z^2 + 4Z + 3)^{-1}$ by Laurent's Series valid for 6
- a) $|Z| < 3$
 b) $|Z| < 1$
 c) $|Z| > 3$

OR

8. a) Find the value of $\oint_C \frac{(12Z-7)}{(Z-1)^2(2Z+3)} dz$ 6
- by using Residue theorem, where C is a circle $|Z| = 2$.
- b) Evaluate $\int_0^{2\pi} \frac{1}{5+3\cos\theta} d\theta$ by contour integration. 6
- c) Evaluate $\int_0^{\infty} \frac{x \sin x}{x^2 + a^2} dx$ by contour Integration. 6

9. a) Solve : 7
 $(mz - ny)p + (nx - \ell z)q = \ell y - mx$
 where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$
- b) Solve : 7
 $(D^2 + 2DD' - 8D'^2)z = e^{2x+y} + \sqrt{2x+3y}$
 where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$
- OR**
10. a) Solve using method of separation of variables, 7
 $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that
 $u = 3e^{-y} - e^{-5y}$ when $x = 0$.
- b) Solve using Laplace transform method : 7
 $\frac{\partial U}{\partial t} + x\frac{\partial U}{\partial x} = x,$
 $x > 0, t > 0, U(x,0) = 0, U(0,t) = 0$
11. a) Investigate the linear dependence of vectors 6
 $X_1 = [1, 1, 1, 3], X_2 = [1, 2, 3, 4], X_3 = [2, 3, 4, 7]$
 and if possible find the relation between them.
- b) Find the Modal Matrix for the matrix 6
 $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$
- c) Use Sylvester's theorem to show that 6
 $2\sin A = (\sin 2)A$
 where $A = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$
- OR**
12. a) If $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as Linear polynomial of A. 6
- b) Solve $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 10y = 0$, given $y(0) = 3, y'(0) = 15$ by Matrix Method. 6
- c) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy + 4zx - 2yz$ to the canonical form by an orthogonal transformation. 6

