W221222

RASHTRASANT TUKADOJI MAHARAJ NAGPUR UNIVERSITY, NAGPUR

SHRI MATHURADAS MOHOTA COLLEGE OF SCIENCE, NAGPUR

Bachelor of Science (B.Sc.) Semester—I

MATHEMATICS Paper – I

M1 -Elementry Mathematics

Time [Three Hours] [Full Marks]

N.B.:- (1) All questions are compulsory and carry equal marks

(2) Questions 1 to 4 have an alternative. Solve each question in full or its alternative in full

(3) Question No. 5 has no alternative and contains eight sub-questions

UNIT- I

Q 1 (A) State and prove De'Moivre's theorem for (i) Positive integer, (ii) Negative integer. (6

(B) Prove that $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1} \cos\left(\frac{n\pi}{4}\right)$, if n is a positive integer (6

OR

(C) Solve the equations $z^2 + 1 = \sqrt{3}i$. (6)

(D) Prove that *if* $\tanh w = z$, *then* $w = \tanh^{-1} z = \frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$. (6)

UNIT- II

Q.2 (A) Find the non-singular matrices P & Q so that PAQ is in normal form for the

Matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$. (6)

(B) Show that the equations : $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$, $x - y + z = -1$ are consistent and solve them (6)

OR

(C) Find the eigen values and associated eigen vectors for the matrix

$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ (6)

(D) Verify Cayley – Hamilton theorem for the matrix

$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$ And hence compute A^{-1} (6)

UNIT- III

Q. 3 (A) Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ may in A.P.

and hence solve $x^3 - 12x^2 + 39x - 28 = 0$ (6)

(B) Solve the reciprocal equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ (6)

OR

(C) Solve the equation $x^3 - 15x = 126$, by Cardan's Method (6)

(D) Solve $x^4 - 3x^3 - 42x - 40 = 0$ by Ferrari's Method (6)

UNIT- IV

Q. 4 (A) Let a and b be integers, not both zero. Then prove that there exists integers x and y Such that $\gcd(a, b) = xa + yb$. (6)

(B) For positive integers a and b prove that $\gcd(a, b) . lcm(a, b) = a . b$ (6)

OR

(C) Solve the linear Diophantine equation $172x + 20y = 1000$ (6)

(D) Solve the system $7x + 3y \equiv 10(mod 16)$ and $2x + 5y \equiv 9(mod 16)$ (6)

Q 5 (A) Prove that $cosh^2 z - sinh^2 z = 1$ $(1\frac{1}{2})$

(B) Find all values of ln (-3) $(1\frac{1}{2})$

(C) Under what condition, the rank of the matrix $A = \begin{vmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{vmatrix}$ is 3 $(1\frac{1}{2})$

(D) Investigate the values of λ and μ, so that the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have no solution $(1\frac{1}{2})$

(E) Find the least possible number of imaginary roots of the equation

$$x^9 - x^5 + x^4 + x^2 + 1 = 0 \quad (1\frac{1}{2})$$

(F) Form the equation whose roots are 1, $1 + i\sqrt{3}$ $(1\frac{1}{2})$

(G) Prove that $a|b$ and $b|a$ imply $a = \pm b$ $(1\frac{1}{2})$

(H) Find $gcd\ (306,657)$ $(1\frac{1}{2})$