B.Sc. (Semester–II) Examination

MATHEMATICS

(M_3–Geometry, Differential & Difference Equations)

Compulsory Paper—I

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Obtain the equation of the sphere circumscribing the tetrahedron whose faces are :

$x = 0, y = 0, z = 0, \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$ 6

(B) Prove that the circles :

$x^2 + y^2 + z^2 - 2x + 3y + 4z - 5 = 0,$

$5y + 6z + 1 = 0;$

$x^2 + y^2 + z^2 - 3x - 4y + 5z - 6 = 0,$

$x + 2y - 7z = 0;$

Lie on the same sphere and find it's equation. 6

OR

(C) Find the equation of the right circular cone whose vertex is the origin and whose axis is the line $x/l = y/m = z/n$ and the semivertical angle θ. 6

(D) Find the equation of the right circular cylinder of radius 2, whose axis passes through the point (1, 2, 3) and has direction cosines proportional to (2, –3, 6). 6

UNIT—II

2. (A) Prove that the necessary and sufficient condition for the ordinary differential equation :

$$M(x, y)\,dx + N(x, y)\,dy = 0$$

to be exact is :

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$ 6

(B) Solve the linear equation :

$$\left(x^2+1\right)\frac{dy}{dx} + 2xy = \sqrt{\left(x^2+4\right)}.$$ 6

OR

(C) Solve $4y^2p^2 + 2pxy\,(3x + 1) + 3x^3 = 0$, where $p = dy/dx$. 6

(D) Solve $\frac{dy}{dx} + \frac{1}{x} \sin 2y = x^2 \cos^2 y$ by transforming to linear form. 6

UNIT—III

3. (A) Solve $(D^2 - 2D + 4)\, y = e^x \cos x$, where $D = \frac{d}{dx}$. 6

(B) Solve $x^2 \frac{d^2y}{dx^2} + 7x \frac{dy}{dx} + 5y = x^5$. 6

OR

(C) Solve $xy^{(2)} - (2x - 1)\, y^{(1)} + (x - 1)\, y = 0$ for which $y = e^x$ is an integral. 6

(D) Solve $y^{(2)} - 2y^{(1)} + y = x^2 e^x$ by the method of variation of parameters. 6

UNIT—IV

4. (A) Solve

$u_{x+2} - 7u_{x+1} + 12u_x = \cos x$ with $u_0 = 0 = u_1$. 6

(B) Solve $u_{x+2} - 5u_{x+1} + 6u_x = 5^x$. 6

OR

(C) Solve $u_{x+2} - 4u_x = 5.3^x$. 6

(D) Solve $u_{x+2} + u_x = \sin(x/2)$. 6

Question—V

5. (A) Show that the equation :

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ represents a sphere. 1½

(B) If the axis of the right circular cylinder passes through origin with direction ratios (1, 2, 3), then find the equation of axis. 1½

(C) Solve $p = \log(px - y)$, where $p = \frac{dy}{dx}$. 1½

(D) Find the integrating factor of linear differential equation

$\sin x \frac{dy}{dx} + y \cos x = 2 \sin^2 x \cos x$. 1½

(E) Reduce $x^2 \frac{d^2y}{dx^2} - 8x \frac{dy}{dx} + 8y = \log x$ to the linear differential equation with constant coefficients. 1½

(F) Find the particular integral of the equation $(1 - D^2)\, y = x$, where $D \equiv d/dx$. 1½

(G) Form the difference equation corresponding to two parameter family $y = ax^2 - bx$. 1½

(H) Solve $u_{x+2} - 2u_{x+1} + u_x = 0$. 1½