Bachelor of Science (B.Sc.) Semester-III (C.B.S.) Examination

MATHEMATICS (ADVANCED CALCULUS, SEQUENCE & SERIES)

Paper—I

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) (i) Verify Rolle's Theorem for the function $f(x) = x^2 - 6x + 8$ in (2, 4).

(ii) Find 'c' of Cauchy's mean value theorem for the functions $f(x) = x^{-2}$, $\phi(x) = x^{-1}$ in [a, b]. 6

(B) Use Lagrange's Mean Value Theorem to show that $\frac{y-x}{1+y^2} < \tan^{-1}y - \tan^{-1}x < \frac{y-x}{1+x^2}$, $0 < x < y$. Hence deduce that $\frac{\pi}{4}+\frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4}+\frac{1}{6}$. 6

OR

(C) Prove that the function :

$$f(x, y) = \begin{cases} y \sin\left(\frac{1}{x}\right) + x \sin\left(\frac{1}{y}\right), & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

is continuous at (0, 0). 6

(D) Expand :

$f(x, y) = x^2 - y^2 - 5xy$ in the powers of $(x - 1)$ and $(y - 2)$. 6

UNIT—II

2. (A) Find the envelope of the family of curves given by :

$$y = mx + \sqrt{4m^2 + 9},$$

where m is a parameter. 6

(B) Find the minimum value of $x^2 + y^2 + z^2$, given that $ax + by + cz = p$ using Lagrange's Multiplier Method. 6

OR

(C) Discuss the maximum and minimum values of the function $u = x^3 + y^3 - 3xy$. 6

(D) Using Lagrange's Multiplier Method, determine extreme value of $u = xyz$ subject to the condition :

$x + y + z = 1$. 6

UNIT—III

3. (A) Show that the sequence $< x_n >$ given by $x_1 = \sqrt{2}$ and $x_{n+1} = \sqrt{2x_n}$ is monotonic, bounded and converges to 2. 6

(B) If sequences $< x_n >$ and $< z_n >$ converge to l and if $x_n < y_n < z_n$ for every $n \in N$ then prove that the sequence $< y_n >$ converges to l. 6

OR

(C) Show that the sequence $\{x_n\}$ given by :

$$x_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \ldots\ldots + \frac{1}{(n+n)^2}$$

converges to zero, using Sandwich Theorem. 6

(D) Define Cauchy Sequence. Prove that the sequence $\left(\frac{n}{n+1}\right)$ is a Cauchy Sequence. 6

UNIT—IV

4. (A) If the series Σu_n of positive terms is convergent, then prove that $\lim_{n \to \infty} u_n = 0$. Give an example to prove that the converse is not true. 6

(B) Test the convergence of the series :

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots\ldots\ldots\ldots$$

using comparison test. 6

OR

(C) Test convergence of the series :

$$\sum_{n=1}^{\infty} \left(\frac{n^3 + a}{2^n + a}\right)$$

by D'Alembert's ratio test. 6

(D) Show that an infinite series in which the terms are alternately positive and negative is convergent if each is numerically less than the preceding term and $\lim_{n \to \infty} u_n = 0$. 6

Question—V

5. (A) By using $\in - \delta$ technique, prove that :

$$\lim_{(x,y) \to (1,2)} (3x + 2y) = 7.$$ 1½

(B) Show that $\lim_{(x,y) \to (0,0)} \frac{x + y}{x - y}$ does not exist. 1½

(C) Find the envelope of $y = mx + \frac{1}{m}$. 1½

(D) Find stationary points of :

$$u = x^2 - 4xy + 2y^2 + 2x.$$ 1½

(E) Find $n_0 \in N$ such that

$$\left|\frac{n}{n-3}-1\right|<\frac{1}{10}, \forall\ n > n_0.$$ 1½

(F) Evaluate :

$$\lim_{n\to\infty} \frac{2+3.10^n}{3+4.10^n}.$$ 1½

(G) Test the convergence of the series $\sum \frac{1}{n^n}$ using the Root test. 1½

(H) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ by Cauchy's Integral Test. 1½