Bachelor of Science (B.Sc.) Semester—III (C.B.S.) Examination

MATHEMATICS (DIFFERENTIAL EQUATIONS & GROUP HOMOMORPHISM)

Paper—II

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Solve all the ***five*** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Prove that :

$$\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x).$$ 6

(B) If λ_j and λ_k are the roots of equation $J_n(\lambda_a) = 0$, then prove that :

$$\int_0^a xJ_n(\lambda_j x)J_n(\lambda_k x)\,dx = 0, \text{ when } j \neq k.$$ 6

OR

(C) Using generating function for $P_n(x)$, prove that :

(i) $P_{2n}(0) = (-1)^n \dfrac{2n!}{2^{2n}(n!)^2}$

(ii) $P_{2n+1}(0) = 0.$ 6

(D) Prove tha recurrence formula :

$nP_n = xP'_n - P'_{n-1}.$ 6

UNIT—II

2. (A) If $Lf(t) = F(s)$, then prove that :

$$L\left[\int_0^t f(u)\,du\right] = \frac{F(s)}{s}. \text{ Hence evaluate } L\left[\int_0^t e^{-u}\cos u\,du\right].$$ 6

(B) Find $L^{-1}\left[\dfrac{s+1}{s^3-5s^2+4s}\right]$. 6

OR

(C) Find $L\left[\dfrac{\cos at - \cos bt}{t}\right]$. 6

(D) Find $L^{-1}\left[\dfrac{s^1}{(s^2+1)^2}\right]$ by convolution theorem. 6

UNIT—III

3. (A) Solve $y'' + 4y' + 3y = e^{-t}$, given that :

$y(0) = 1,\ y'(0) = 1.$ 6

(B) Solve $ty'' + y' + ty = 0$, given that :

$y(0) = 1,\ y'(0) = 0.$ 6

OR

(C) Solve $\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}$, given that :

$u(0, t) = 0,\ u(5, t) = 0,\ u(x, 0) = \sin \pi x.$ 6

(D) Find the Fourier sine transform of :

$\frac{e^{-\lambda x}}{x}, x > 0.$ 6

UNIT—IV

4. (A) Prove that a subgroup N of a group G is a normal subgroup of G if and only if each left coset of N in G is a right coset of N in G. 6

(B) Prove that the generator of a cyclic group of order n are all the elements a^p, p being prime to n and $0 < p < n$. Hence find which elements of the group :

$G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$ can be used as generators of the group G. 6

OR

(C) If f is a homomorphism from group G into group G′, then prove that kernel K of f is a normal subgroup of G. 6

(D) Given that (I, +) is a group of integers and G = {1, –1, i, –i} is a multiplicative group. Show that $f : I \rightarrow G$ defined by $f(x) = i^x,\ \forall x \in I$ is a homomorphism. Also find the kernel of f. 6

QUESTION—V

5. (A) Show that $\int_0^x x^2 J_0 J_1\, dx = \frac{1}{2}x^2 J_1^2$. 1½

(B) Evaluate $\int_0^1 x^2 P_2(x) dx.$ 1½

(C) Using definition of Laplace transform, show that $L[e^{at}] = \frac{1}{s-a}$, $s > a$. 1½

(D) Evaluate $L^{-1}\left[\frac{s}{(s+4)^2}\right]$. 1½

(E) Show that $L\left(\frac{\partial u}{\partial x}\right) = \frac{dU}{dx}$,

where $U = U(x, s) = L[u(x, t)]$. 1½

(F) Show that $F[e^{ics} f(x)] = \hat{f}(s+c)$, where F is a Fourier transform of f(x). 1½

(G) Find quotient group G/N if N = {1, –1} is a normal subgroup of a multiplicative group G = {1, –1, i, –i}. 1½

(H) Let G and G be multiplicative groups and a mapping $f : G \to G'$ be defined by $f(x) = x^2$, $\forall\ x \in G$. Find whether f is an isomorphism. 1½