TKN/KS/16 – 5839

Third Semester B. Sc. Examination

STATISTICS

Paper – I

(Statistical Methods)

Time : Three Hours] [Max. Marks : 50

N. B. : All the **five** questions are compulsory and carry equal marks.

1. (A) Define

   (i) Joint p. d. f. of two random variables X and Y.

   (ii) Marginal densities of X and Y.

   (iii) Covariance between two random variables X and Y.

   (iv) Stochastic independence of random variables X and Y.

   The fraction X of male runners and the fraction Y of female runners who compete in marathon races is described by the joint density function

$$f(x, y) = \begin{cases} 8xy, & 0 < x < 1,\ 0 < y < x \\ 0, & \text{elsewhere} \end{cases}$$

   Find (i) marginal densities of random variables X and Y.

   (ii) E (X) , E (Y) , E (XY)

   (iii) Cov (X , Y)

   (iv) Check the independence of X and Y. 10

OR

(E) Let f (x , y) be joint p. d. f. of r. Vs. X and Y and M (t_1, t_2) be mgf. of the joint probability distribution.

Let h (x) and g (y) be functions of x alone and y alone respectively. Show that the r. Vs. X and Y are independently distributed iff

(i) b (x , y) = h (x) . g (y)

(ii) $M(t_1, t_2) = M(t_1, 0) \cdot M(0, t_2)$.

Let Xt_1 and X_2 be two independent random variables resulting from two throws of an unbiased die. Find the p. m. f. of $y = X_1 + X_2$.

Also find E (Y) and V (Y). 10

2. (A) Derive the p. m. f. of trinomial distribution. According to a genetics theory, a certain cross guinea pigs will result in red, black and white offsprings in the ratio 8 : 4 : 4. Find the probability that among 8 offsprings 5 will be red, 2 black and 1 white. 10

OR

(E) If the random variables X and Y follow bivariate normal distribution with parameters μ_1 , μ_2 , σ_1 , σ_2 and ϱ, then find marginal p. d. f. of r. v. X. Also find conditional density of Y given X. Hence state conditional mean and conditional variance of Y given x. 10

3\. (A) Define a random sample. Write the steps for drawing a random sample of size 10 from Binomial population with parameters σ and 0.3.

(B) Let X be a continuous random variable with probability distribution

$$f(x) = \begin{cases} x/12 & , 1 < x < 5 \\ 0 & , \text{elsewhere} \end{cases}$$

find the probability distribution of the random variable $y = 2x - 3$.

(C) Let X_1 and X_2 be random variables with joint probability distribution

$$p(x_1, x_2) = \begin{cases} \dfrac{x_1 x_2}{18} & , x_1 = 1, 2 ; x_2 = 1, 2, 3 \\ 0 & , \text{elsewhere} \end{cases}$$

Find the probability distribution of $Y = X_1 \cdot X_2$.

(D) The hospital period, in days, for patients following treatment for a certain type of kidney discorder is a random variable Y = X + 4, where X has the density function

$$f(x) = \begin{cases} \dfrac{32}{(x+4)^3} & , x > 0 \\ 0 & , \text{elsewhere} \end{cases}$$

Find (i) the p. d. f. of Y

(ii) the probability that the hospital period for a patient following the treatment will exceed 8 days. $2\frac{1}{2} \times 4 = 10$

OR

(E) X and Y are independent Gamma variates with parameters (α, λ) and (β, λ) respectively. Show that V = X + Y follows Gamma distribution with parameters $(\alpha + \beta, \lambda)$ and

$$V = \frac{X}{X+Y}$$

follows Beta distribution with parameters (α, β).

(F) If $X \sim N(\mu, \sigma^2)$ then show that $Y = a + bX, b \neq 0$ is also normally distributed with mean $a + b\mu$ and variance $b^2 \sigma^2$. Also if a random sample of size n is taken from $N(\mu, \sigma^2)$ then find the distribution of sample mean $\overline{X}$. 5 + 5

4\. (A) Define Chi – square statistic. State the p. d. f. of Chi – square distribution. Find its m. g. f. and hence find mean and variance of this distribution. Also find mode of Chi – square distribution. 10

OR

(B) Define Fisher's t statistic. Derive p. d. f. of t distribution. Show that mean of t distribution is zero. Find its variance. 10

5\. Solve any **ten** of the following :—

(A) State additive property of Chi – square distribution.

(B) State p. d. f. of F distribution.

(C) If X has t distribution with n d. f., derive the distribution of χ^2.

(D) Define correlation coefficient and state its limits.

(E) Define conditional mean of a discrete random variable X given Y = y.

(F) If the pair of random variables (X , Y) follows Bivariate normal distribution with the parameters

$\mu_x = 10$, $\sigma_x^2 = 9$, $\mu_y = 12$, $\sigma_y^2 = 16$

$\varrho = 0.6$. Find E (Y / x = 2)

(G) State m. g. f. of trinomial distribution.

(H) If the joint p. m. f. of random variables X and Y is

$$f(x, y) = \frac{x + y}{21}, \quad x = 1, 2, 3 \text{ and } y = 1, 2$$

find marginal distribution of X.

(I) If r. Vs. X and Y follow Bivariate normal distribution with $\varrho = 0$ show that X and Y are independent.

(J) Define a statistic.

(K) If $X_1 \sim B(n_1, p)$ and $X_2 \sim B(n_2, p)$, state the probability distribution of $X_1 + X_2$.

(L) Let X have a p. m. f.

$f(x) = \frac{1}{4}$, x = 0 , 1 , 2 , 3

Find the p. m. f. of y = 3x. 1×10=10