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Bachelor of Science (B.Sc.) Semester—IV (C.B.S.) Examination

MATHEMATICS (Mechanics)

Paper—II

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Forces P, Q, R act along the sides BC, CA, AB of a triangle ABC and forces P_1, Q_1, R_1 act along OA, OB, OC where O is the centre of the circumscribing circle. Prove that if the six forces are in equilibrium, then :

$$P \cos A + Q \cos B + R \cos C = O \text{ and } \frac{PP_1}{a} + \frac{Q \cdot Q_1}{b} + \frac{R \cdot R_1}{C} = 0 .$$ 6

(B) Five weightless rods of equal length are joined together so as to form a rhombus ABCD with one diagonal BD. If a weight W be attached to C and the system be suspended from A. Show that there is a thrust in BD equal to $W/\sqrt{3}$. 6

OR

(C) A uniform chain of length ℓ is stretched between two points in the horizontal line such that the maximum tension is equal to n times its weight. Show that the least possible sag in the middle is :

$$\ell\left[n - \sqrt{n^2 - \frac{1}{4}}\right].$$ 6

(D) If a chain is suspended from two points A, B on the same level, and depth of the middle point below AB is (ℓ/n), where 2ℓ is the length of the chain, then show that the horizontal span **AB** is equal to :

$$\ell\left(n - \frac{1}{n}\right)\log\left(\frac{n+1}{n-1}\right).$$ 6

UNIT—II

2. (A) If the radial and transverse velocity of a particle are always proportional to each other then :

(i) Show that the path is an equiangular spiral

(ii) If in addition, the radial and transverse acceleration are always proportional to each other, show that the velocity of the particle varies as some power of the radius vector. 6

(B) If the tangential and normal accelerations of a particle describing a plane curve to be constant throughout, prove that the radius of curvature at any point is $(at + b)^2$. 6

OR

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(C) At the ends of three successive seconds the distances of a point moving with S.H.M. from its mean position measured in the same direction are 1, 5 and 5. Show that the period of a complete oscillation is :

$2\pi/\cos^{-1}(3/5)$. 6

(D) In S.H.M., at what distance from the centre will the velocity be half of the maximum ? 6

UNIT—III

3. (A) State and prove D'Alembert's principle for a mechanical system of particles. 6

(B) (i) Construct the Lagrangian for a particle moving in space using Cartesian coordinates and then deduce the equations of motion.

(ii) A bead is sliding on a uniformly rotating wire in a force free space. Show that the acceleration of the bead is $\ddot{r} = r\omega^2$, where ω is the angular velocity of rotation. 6

OR

(C) If L is a Lagrangian for a system of n degrees of freedom satisfying Lagrange's equations, show by direct substitution that $L' = L + \frac{dF}{dt}$ also satisfies Lagrange's equations, where $F = F(q_1, q_2, ..., q_n, t)$ is any arbitrary but differentiable function of its arguments. 6

(D) Define : Rayleigh's dissipation function R. Show that the rate of energy dissipation due to friction is twice the Rayleigh's dissipation function R. 6

UNIT—IV

4. (A) Prove that the problem of motion of two masses interacting only with one another always be reduced to a problem of the motion of a single mass. 6

(B) If the potential energy is a homogeneous function of degree–1 in the radius vector $\vec{r}_i$, then prove that the motion of a conservative system takes place in a finite region of space only if the total energy is negative. 6

OR

(C) Consider a system in which the total forces acting on the particle consist of conservative forces $\vec{F}_i'$ and the frictional forces $\vec{f}_i$ proportional to velocity. Show that for such a system the virial theorem holds in the form $\overline{T} = -\frac{1}{2}\overline{\sum_i \vec{F}_i' \circ \vec{r}_i}$. 6

(D) A particle describes the equiangular spiral $r = ae^{\theta \tan \alpha}$ under a force to the pole. Find the law of force. 6

QUESTION—V

5. (A) Define a couple and moment of a couple. 1½

(B) For a common catenary, prove that :

(i) $y^2 = c^2 + s^2$

(ii) $y = c \sec \psi$. 1½

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(C) Let $\vec{r}$ be the position vector of a particle P with respect to the origin O. Let $\hat{r}$ and $\hat{\theta}$ be the unit vectors in radial and transverse direction. Prove that :

$$\frac{d\hat{r}}{dt} = \dot{\theta}\,\hat{\theta} \text{ and } \frac{d\hat{\theta}}{dt} = -\dot{\theta}\,\hat{r}\,.$$ 1½

(D) For the displacement x = a cos nt + b sin nt in S.H.M., show that $\mu = n^2$. 1½

(E) Prove the relation $\dot{\vec{L}} = \vec{N}$ and hence prove $\vec{L}$ is conserved if $\overline{N} = 0$. 1½

(F) Define :

(i) Degree of freedom

(ii) Equation of constraint for a particle in motion. 1½

(G) Prove that the path of a particle in a central force field lies in one plane. 1½

(H) If the conservative force F is given by $F = \frac{k}{r^2}$, then find the potential V.. 1½