TKN/KS/16/5853

Bachelor of Science (B.Sc.) Semester—IV (C.B.S.) Examination

MATHEMATICS

Paper—I

(M_7—Partial Differential Equation and Calculus of Variation)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Questions **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Find the integral curves of the equations :

$$\frac{dx}{y(x+y)-bz}=\frac{dy}{x(x+y)+bz}=\frac{dz}{z(x+y)}.$$ 6

(B) Verify the equation :

$x(y^2 - a^2)\,dx + y(x^2 - z^2)\,dy - z\,(y^2 - a^2)\,dz = 0$ is integrable and solve it. 6

OR

(C) Verify that the equation :

$(y^2 + yz)\,dx + (z^2 + xz)\,dy + (y^2 - xy)\,dz = 0$

is integrable and find its solution. 6

(B) Find P.I. of

$(D^2 - 6DD' + 9D'^2)z = 12x^2 + 36xy$

where $D = \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}$. 6

OR

(C) Solve :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - z = \cos(x + 2y).$$ 6

(D) Solve :

$$x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} = x^2 y,$$ by using $x = e^u$ and $y = e^v$. 6

UNIT—IV

4. (A) Prove a necessary condition for the functional

$$I[y(x)] = \int_{x_0}^{x_1} F(x, y, y')\,dx$$ to be an extremum is that

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0.$$ 6

(B) Find the shortest curve joining the two points (x_1, y_1) and (x_2, y_2) by using the functional

$$I[y(x)] = \int_{x_1}^{x_2} \sqrt{[1 + (y')^2]}\,dx$$

with $y(x_1) = y_1$ and $y(x_2) = y_2$. 6

OR

(C) Find the extremum for the functional

$$I[y(x)] = \int_0^{\pi} (16y^2 - y'^2 + x^2)\,dx;$$

$y(0) = y(\pi) = 0,\ y'(0) = y'(\pi) = 1.$ 6

(D) Write Euler's-Ostrogradsky equation for the functional $$I[z(x, y)] = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right] dxdy.$$ 6

UNIT—V

5. (A) Find the integral curves of

$$\frac{dx}{1} = \frac{dy}{-2} = \frac{dz}{3x^2 \sin(2x + y)}.$$ 1½

(B) Form a partial differential equation by eliminating arbitrary constants from the equation

$z = (x + a)(y + b).$ 1½

(C) Find the complete integral of $pq = 1$, by Charpit's method. 1½

(D) Write the Jacobi's auxiliary equation for $p^2x + q^2y = z$. 1½

(D) Prove a necessary and sufficient condition that there exists between two functions u(x, y) and v(x, y) a relation F(u, v) = 0, not involving x or y explicitly is that $\frac{\partial(u, v)}{\partial(x, y)} = 0$. 6

UNIT—II

2. (A) Find the general solution of the partial differential equation :

$$z(xp - yq) = y^2 - x^2$$ 6

(B) Find the integral surface of the partial differential equation $x^2p + y^2q + z^2 = 0$ through the curve $xy = x + y$, $z = 1$. 6

OR

(C) Using Charpit's method, find the complete integral of the partial differential equation :

$$p = (z + qy)^2.$$ 6

(D) Show that a complete integral of $f\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right) = 0$ is $u = ax + by + \phi(a, b) z + c$, where a,b,c are arbitrary constants and $f(a, b, \phi) = 0$. Further find also the complete integral of

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{\partial u}{\partial x} \cdot \frac{\partial u}{\partial y} \cdot \frac{\partial u}{\partial z}.$$ 6

UNIT—III

3. (A) Solve :

$$(D^2 + DD' - 6D'^2)z = y \sin x$$

where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$. 6

(E) Solve $\frac{\partial^2 z}{\partial x \partial y} = 2x + 2y$ by integrating with respect to x and y. 1½

(F) Find P.I. of $(D^2 - 4DD' + 4D'^2)z = e^{2x + y}$. 1½

(G) Let $I[y(x)] = \int_0^1 [y(x)]^2 . dx$ be a functional.

If $y(x) = \sqrt{1 + x^2}$ then find I [y(x)]. 1½

(H) Find the distance of order zero between the functions $y = x^2$ and $y = x$ on the interval [0, 1]. 1½