Bachelor of Science (B.Sc.) Semester-IV (C.B.S.) Examination

MATHEMATICS (MECHANICS)

Paper–II

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT–I

1. (A) Show that a system of coplanar forces acting upon a rigid body (at different points of it) can be reduced to a single force at any arbitrary point, and a couple whose moment is equal to the sum of the moments of the given force about that point. 6

(B) A regular hexagon ABCDEF consists of six equal rods which are each of weight W and are freely jointed together. The hexagon rests in a vertical plane and AB is in contact with a horizontal table. If C and F be connected by a light string, prove that its tension is $\frac{W}{\sqrt{3}}$. 6

OR

(C) Derive the intrinsic equation s=c tanx of the common catenary. 6

(D) A uniform chain of length ℓ is stretched between two points in the horizontal line such that the maximum tension is equal to n times its weight. Show that the least possible sag in the middle is :

$$\ell\left[n-\sqrt{\left(n^2-\frac{1}{4}\right)}\right].$$ 6

UNIT–II

2. (A) If the path of a moving particle is an equiangular spiral $r=ae^{\theta\cot\alpha}$ and the radius vector to the particle has constant angular velocity, show that the resultant acceleration of the particle makes an angle 2α with the radius vector and is of magnitude v^2/r, if v is the speed of the particle. 6

(B) A particle describes the curve $r=ae^{\theta}$ (equiangular spiral) with a constant angular velocity about the pole O of a spiral. Obtain the radial and transverse accelerations of a particle. 6

OR

(C) A particle is describing a plane curve. If the congential and normal accelarations are each constant throughout the motion, prove that the angle χ, through which the direction of motion turns in time t is given by $\chi=A\log(1+Bt)$. 6

(D) A particle is moving with simple Harmonic motion and while making an excursion from one position of rest to another, its distances from the middle point of its path at three consecutive seconds are observed to be x_1, x_2, and x_3. Prove that the time of complete oscillation is

$$\frac{2\pi}{\cos^{-1}\left(\frac{x_1+x_3}{2x_2}\right)}.$$ 6

UNIT–III

3. (A) State and prove D'Alembert's principle for a system of n particles. 6

(B) Derive Lagrange's equations of motion of a particle in plane using polar coordinates r and θ as

$F_r = m\ddot{r} - mr\dot{\theta}^2$ and $F_\theta = mr\ddot{\theta} + 2m\dot{r}\dot{\theta}$ 6

OR

(C) If L is a Lagrangian for a system of n degrees of freedom satisfying Lagrange's equations, show by direct substitution that :

$L' = L + \left(\frac{dF}{dt}\right)$ also satisfies Lagrange's equations, where $F=F(q_1, q_2, \ldots, q_n, t)$ is any arbitrary but differentiate function of its argument. 6

(D) Define Rayleigh's dissipation function R and show that the rate of energy dissipation due to friction is 2R. 6

UNIT–IV

4\. (A) Prove that the central force motion of the two bodies can always be reduced to an equivalent one body problem. 6

(B) If the potential energy is a homogeneous function of degree –1 in the radius vector $\vec{r}_i$, then prove that the motion of a conservative system takes place in a finite region of space only if the total energy is negative. 6

OR

(C) For a central force field F, prove that the differential equation for the orbit is given by :

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{h^2u^2}F\left(\frac{1}{u}\right), u = \frac{1}{r}$$ 6

(D) Consider a system in which the total forces acting on the particles consist of conservative forces $\vec{F}_i$, and frictional forces $\vec{f}_i$ proportional to the velocity. Show that for such a system the virial theorem holds in the form $\overline{T} = -\frac{1}{2}\overline{\sum \vec{F}_i \cdot \vec{r}_i}$ providing the motion reaches a steady state and is not allowed to die down as a result of frictional forces. 6

QUESTION–V

5\. (A) State the principle of virtual work for the system of forces acting on a particle in equilibrium. 1½

(B) Show that s=c sinh(x/c) for the common catenary. 1½

(C) The velocities of a particle along and perpendicular to the radius vector are λ_r and μ_θ. Show that the equation of path is $\theta = ce^{-\mu/\lambda r}$. 1½

(D) For x=a cosnt + b sinnt in SHM, show that $n^2=\mu$. 1½

(E) Define for a mechanical system or a particle :

(i) Degree of freedom

(ii) Force of constraint. 1½

(F) State Lagrange's equations of motion for conservative holonomic system. 1½

(G) In a central force field, prove that the motion of a particle is in plane. 1½

(H) If a particle moves in an ellipse under a central force directed towards its focus, whose polar equation is $\frac{h}{r} = 1 + e\cos\theta$, where e is the ecentricity and h is a constant. Then find the Law of force. 1½