

Bachelor of Science (B.Sc.) Semester—V (C.B.S.) Examination**MATHEMATICS****Paper—2****(M₁₀-Metric Space, Complex Integration and Algebra)**

Time : Three Hours]

[Maximum Marks : 60]

- N.B. :**— (1) Solve all the *five* questions.
 (2) All questions carry equal marks.
 (3) Question Nos. 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Let $\{E_n\}$, $n = 1, 2, 3, \dots$ be a sequence of countable sets, then prove that $S = \bigcup_{n=1}^{\infty} E_n$ is countable. 6
 (B) Show that the function d defined by $d(x, y) = |x - y| \forall x, y \in R$, is a metric on the set R of real numbers. 6
 (C) Prove that every neighbourhood is an open set. 6
 (D) For any collection $\{F_\alpha\}$ of closed sets prove that $\bigcap F_\alpha$ is closed. Give an example to show that arbitrary union of closed sets is not closed. 6

UNIT—II

2. (A) Prove that closed subsets of compact sets are compact. 6
 (B) If $\{I_n\}$ is a sequence of intervals in R^1 such that $I_n \supset I_{n+1}$ ($n = 1, 2, 3, \dots$), then prove that $\bigcap_{n=1}^{\infty} I_n$ is not empty. 6
 (C) Prove that every convergent sequence in a metric space is a Cauchy sequence. 6
 (D) Prove that every bounded infinite subset R^k has a limit point in R^k . 6

UNIT—III

3. (A) Show that a commutative ring D is an integral domain if and only if for $a, b, c \in D$ with $a \neq 0$, the relation $ab = ac$ implies $b = c$. 6
- (B) If R is a ring and L is a left ideal of R .
Let $\lambda(L) = \{x \in R / xa = 0 \text{ for all } a \in L\}$. Prove that $\lambda(L)$ is a two-sided ideal of R . 6
- OR
- (C) If ϕ is a homomorphism of R into R' with Kernel $I(\phi)$ then prove that:
 (i) $I(\phi)$ is a subgroup of R under addition.
 (ii). If $a \in I(\phi)$ and $r \in R$ then both $ar, ra \in I(\phi)$. 6
- (D) If U is an ideal of a ring R , then prove that R/U is homomorphic to R . 6

UNIT—IV

4. (A) Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ using Cauchy's integral formula where C is the circle $|z| = \frac{3}{2}$. 6
- (B) Verify Cauchy's theorem for $\int_C z^3 dz$ over the boundary of the triangle with vertices $(1, 2), (1, 4), (3, 2)$. 6
- OR
- (C) State and prove Cauchy's Residue theorem for an analytic function. 6
- (D) Evaluate the residue of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at $1, 2, 3$ and infinity and show that their sum is zero. 6

Question—5

5. (A) Prove that $\left(\bigcup_{\alpha} G_{\alpha}\right)^c = \bigcap_{\alpha} G_{\alpha}^c$. 1½
- (B) Define almost countable and uncountable sets. 1½
- (C) If $A = [0, 1]$ and $B = (1, 2)$, then show that A and B are not separated in R . 1½
- (D) Define K-cell and explain 2-cell. 1½
- (E) Let R be a ring and $a, b \in R$. Prove that $(a + b)^2 = a^2 + ab + ba + b^2$. 1½
- (F) If ϕ is a homomorphism of ring R into R' , then show that $\phi(0) = 0' \in R'$. 1½
- (G) Evaluate $\int_C \frac{1}{z^2 - 1} dz$, where C is the circle $x^2 + y^2 = 4$. 1½
- (H) Evaluate $\int_0^{1+i} (x - y + ix^2) dz$ along the straight line from $z = 0$ to $z = 1 + i$. 1½

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