Bachelor of Science (B.Sc.) Semester—VI Examination

M_{12} : SPECIAL THEORY OF RELATIVITY

Optional Paper—2

(Mathematics)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) A particle of mass m_1 moving with velocity u makes a head on collision with a particle of mass m_2 initially at rest, so that their final velocities v_1 and v_2 are along the same line. If the collision is elastic, show that : $v_2 = \dfrac{2u}{1+\dfrac{m_2}{m_1}}$. 6

(B) Show that :

$x^2 + y^2 + z^2 - c^2t^2$ is Lorentz invariant. 6

OR

(C) Prove that :

(i) Newtonian fundamental equations of Motion and

(ii) Length of a rod

are invariant under Galilean transformations. 6

(D) Give geometric interpretation of Lorentz transformations. 6

UNIT—II

2. (A) Explain the phenomenon of ''Time dilation'' in special theory of relativity. Also find at what speed should a clock be moved so that it may appear to lose 1 minute in each hour. 6

(B) Show that if l_o is the rest volume of a cube, then $\ell_0^3\sqrt{1-\dfrac{v^2}{c^2}}$ is the volume viewed from a reference frame moving in a direction parallel to an edge of the cube. 6

OR

(C) Let q and q' be the velocities of a particle in two inertial systems S and S' respectively. Initially systems are coincident and S' has velocity v relative to S in X-direction. Show that :

$$q^2 = \frac{q'^2+v^2+2q'v\cos\theta'-\left(q'\dfrac{v\sin\theta'}{c}\right)^2}{\left(1+\dfrac{v}{c^2}q'\cos\theta'\right)^2}$$ 6

(D) A particle instantaneously at rest in frame of reference S', experiences an acceleration in it represented by a vector $\vec{f}' = 3\vec{i} + 4\vec{j} + 12\vec{k}$. What is the acceleration measured from the frame of reference S; given that S' moves with velocity 0.98C relative to S along positive x-axis ? 6

UNIT—III

3. (A) For the symmetric tensor $A^{\mu\nu}$, prove that :

(i) Symmetric property for $A^{\mu\nu}$ remains unchanged under tensor law of transformation, and

(ii) $A^{\mu\nu}$ has 10 independent components in 4-dimension. 6

(B) If g_{ij} is a symmetric tensor, then define its conjugate g^{ij} and prove that g^{ij} is a tensor. 6

OR

(C) Transform $ds^2 = dx^2 + dy^2 + dz^2$ in spherical coordinates. 6

(D) If A^{μ} and B^{μ} are contravariant vectors and $C_{\mu\nu}\, A^{\mu}\, B^{\nu}$ is an invariant then prove that $C_{\mu\nu}$ is a tensor of the second order. 6

UNIT—IV

4. (A) Derive transformation equations for momentum and energy under Lorentz transformations. 6

(B) Explain the formulation of energy momentum four vector p^{μ} to obtain $p^{\mu} = (p_x, p_y, p_z, \frac{E}{C})$. 6

OR

(C) Derive component form of Maxwell's equations of electromagnetic theory in vacuum. 6

(D) Derive transformation equation for mass of the particle. 6

Question—V

5. (A) Explain, how did Einstein rule out the concept of absolute time. 1½

(B) Let S and S' be inertial frames and S' is moving with velocity $v = (o, v_y, o)$ along Y-axis. Then write the form of Lorentz transformation equations. 1½

(C) An electron is moving with a speed of 0.8c in a direction opposite to that of a moving photon. Calculate the relative velocity of electron and photon. 1½

(D) The length of a rocketship is 100 meters on the ground. When it is in flight its length observed on the ground is 81 meters, calculate its speed in terms of speed of light c. 1½

(E) Write transformation law for the tensor quantity $A^{ij}_{\alpha\beta\gamma}$ and find its rank when $\alpha = i$. 1½

(F) Minkowskian metric is $ds^2=(dx^4)^2 -(dx^1)^2 -(dx^2)^2- (dx^3)^2$, so find $g = |g_{ij}|$. 1½

(G) Using $p^2 - \frac{E^2}{c^2} = -M_o^2c^2$, prove that a particle of zero rest mass travels with the speed of light c. 1½

(H) Derive Newtonian Kinetic energy of the particle from its expression of relativistic kinetic energy. 1½